Medical Mathematics for Medics

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Ah, the joy of med math. The first thing to understand about med math is that all of the teachers you had through middle and high school were right – you did need to know all that crap for when you got out into the real world. Med math is the perfect example of that. Yes, there are simple formulas that exist for each type of calculation that we do in medicine, but memorizing those formulas, keeping a field guide with each one written down, or even storing them on a PDA or using a medical calculator is unacceptable – if you're the one dosing up meds, you need to understand what you are doing, or what your formula is doing, or what that medical calculator you have on your PDA is doing. Field guides get left behind or bloodied or lost; PDA batteries run dry. What are you going to do when your patient crumps on you, you're all alone, and it's down to you and the vial of medicine that you know he so desperately needs?

Furthermore, if there is one thing (other than pay scale) that separates medics from nurses, it's the ability to do med math. Yes, it's true, some nurses who use pumps and calculators on a daily basis have long since forgotten how to do med math (I, in fact, have witnessed this first-hand multiple times), but med math is still the main difference between the two professions, and here's why: while nurses must achieve a minimum passing score on a med math test before being allowed to progress past a certain level in nursing school, there is no such standard or hurdle for medics. Contrast the fact that while nurses had to achieve a minimum passing score on a med math test at some point in their education, it's entirely possible to flunk the math portion of a NREMT-P curriculum, miss every single math question on the registry, and still walk around wearing a glitter patch. And trust me, that scenario has actually played out in reality on more than one occasion – there are medics walking around out there who are allowed to give a great number of meds, but they have no idea how much medication to give. Some of them *might* be able to plug a formula into a calculator (and some might not be able to do even that), but without the formulas and the calculator, they're completely useless. Where's the sanity in that? If you have to break out the calculator to figure how many milliliters of succinylcholine to push during the middle of an RSI, you're simply doing the patients, and the profession, a disservice.

Preamble

As we discussed in the introduction, there are already a great many ways to perform medical calculations, or rather, have them performed for you. PDA software, drip charts, dose tables, and even formulas. And while these are effective, easy methods of getting work done, if **you** are going to be the one giving medications to patients and deciding the doses to give, then **you** need to understand how the computer or the chart arrived at its answer. So, while this article will cover the what and the how of medical mathematics, it most importantly discusses the **why** of medical mathematics. It's important to understand what to calculate and how to calculate it, and software and charts are great for doing that, but what you really need to take away from all of this is **why**? So for each calculation we do, we're going to illustrate the known information, followed by the desired, or goal, information. This will hopefully instill in you an understanding of **why** each calculation is performed. As you gain proficiency with this process, medical mathematics will become automatic, even second-nature. After enough practice, you will eventually be able to perform most, if not all, of the calculations in your head, and in true emergencies, this is what is needed – no breaking out a computer or searching frantically for a chart, but rather, a calmness, where in the space of a few seconds, you look calmly at the medication you need to give, think your way through the dosing, recheck your solution, and administer the medication.

Preliminary Things

Before we begin there are a few things we need to review. Of all the variables in medical mathematics, few are more important than mass, weight, and volume.

Mass

Mass refers to the amount of medication in a sense of its weight, for mass and weight are almost synonymous on Earth. Medications are weighed accurately, and then dissolved in solution, compressed with other chemicals into pill form, or otherwise stored in a manner suitable for accurate administration to patients. When giving medications to patients, we typically want to give a specific mass of drug to the patient, be it so many milligrams (mg) or so many micrograms (mcg) or even so many grams (g). Because most medications are dissolved in liquid, the way that we give a specific mass of drug to a patient is by giving the patient a specified volume of liquid containing a known concentration of drug.

Volume

Volume simply refers to how much space something takes up, or how much space a vessel has to be filled. In medical mathematics, virtually everything centers on the milliliter, or one one-thousandth (0.001) of a liter. As discussed above, most medicines contain a known mass of drug dissolved in a known volume of liquid. Again, since virtually all of medical mathematics centers on the milliliter, abbreviated mL, we typically deal with how much mass of a drug is dissolved in how many mL's of drug. For instance, epinephrine 1:1000 contains 1 mg (milligram) of epinephrine dissolved in 1 mL of liquid, which is typically water or saline. For clarity, I'll also mention than a milliliter (mL) is the same as a cubic centimeter (cc)

Concentration

Concentration is simply a ratio of the two variables we have discussed, mass and volume. Again, epinephrine 1:1000 contains 1 mg of epinephrine in 1mL of fluid, so the concentration of 1:1000 epinephrine is simply 1mg/1mL, often just abbreviated to 1 mg/mL.

Metric Units

Our system of measurement is based on English units – miles, pounds, feet, etc. The rest of the world (everyone but Britain, Australia, the U.S., the Falkland Islands, and a few others…Britain, Australia, and even the Falkland Islands have gradually switched over to the metric system, and the U.S. is really the only "English" country left today) uses the metric system. More importantly, virtually the entirety of medicine is based on the metric system. To make things even stranger, abbreviations for English units are usually followed by a period, whereas metrics are not (e.g., mi. vs. km). The table below provides the relationships that we will need:

Measurement	English Units	Metric Units
Distance	1 mile (mi.)	1.6213 kilometers (km)
Distance	0.625 mi.	1 km
Weight	1 pound (lb.)	0.454 kilograms (kg)
Weight	2.2 pounds (lbs.)	1 kilogram (kg)
Volume	1 gallon	3.79 liters (L)
Volume	0.264 gal.	1 L
Length	1 inch (in.)	2.54 centimeters (cm)
Length	0.39 in.	1 cm

Prefixes

Turning now completely to the metric system, we have the issue of prefixes to deal with. These are smaller and larger variations of the standard units (liters [L], meters [m], grams [g]), and they are based on powers of 10 – usually 10^3, or 10*10*10, or 1,000 – and they are named as follows:

Prefix	Value	Exponent
giga (G)	x1,000,000,000	10^9
mega (M)	x1,000,000	10^6
kilo (K or k)	x1,000	10^3
milli (m)	x$1,000^{-1}$ or 0.001	10^{-3}
micro (**m** or mc)	x$1,000,000^{-1}$ or 0.000001	10^{-6}
nano (n)	x$1,000,000,000^{-1}$ or 0.000000001	10^{-9}

And here the X^{-1} is simply the inverse, i.e., 1/X, so for example milli- means 1/1,000 (that is to say, one one-thousandth of something, so a milligram would be equal to one one-thousandth of a gram).

Turning our attention to medical mathematics, the most common metric units we have to deal with are the kilogram (kg), usually used to describe a patient's weight, the milliliter (mL), usually used to describe volume, though liters (L) are used as well; mass of medication is most commonly described in milligrams (mg), though micrograms (formerly '**m**g', but now 'mcg' thanks to the Joint Commission[1]), and grams (simply g) are not at all uncommon.

[1] I.E., the Joint Commission for the Accreditation of Healthcare Organizations, or JCAHO

Dimensional Analysis

Dimensional analysis is easily both the most poorly understood and the most powerful mathematical tool that a paramedic can possess. I cannot underscore the importance of mastering this technique, mainly for two reasons: (1) it is the only way to truly verify that your calculation is correct, and (2) if (err…when) you find yourself completely lost with no idea whatsoever what to calculate next, dimensional analysis is an unwavering guide – simply arrange things so that the everything in excess cancels, and you are left with whatever you are trying to solve for.

For the uninitiated, that last little quip was a hint as to what dimensional analysis *is* – it is simply a method of arranging fractions or ratios of measure to eliminate things that should not be in our final solution. I have a feeling that things are getting a little abstract here, so I will quickly move to an example, but before I do I need to review a little algebra, so bear with me.

There are really only three things we need to review from algebra: (1) numerators and denominators, (2) the fact that anything divided by itself is simply 1, and (3) any number X multiplied by 1 is simply X. Allow me to demonstrate these principles mathematically:

A fraction, or ratio, is simply a description of the relationship between two numbers. The number "on top" is the numerator, and the number "on the bottom" is the denominator. So if we had the value 0.5, or 50%, or ½, we could also write it

$$\frac{1}{2} \rightarrow \frac{numerator}{denominator},$$

and that's about all there is to it.

Moving on to my next review topic, anything divided by itself is 1. So 2/2=1, 3/3=1, and more to the point for medical mathematics, any time you have the same units in both the numerator and the denominator, they also divide out to equal 1 – for example, mg/mg=1, kg/kg=1, etc. This property of non-numeric quantities equating to 1, i.e., mg/mg=1, is more commonly described as "canceling each other out."

And lastly, quantity multiplied by 1 is simply the initial quantity: 1*1=1, 2*1=2, 300*1=300; and again for medical mathematics, this also applies to the units of measure – e.g., mg*1=mg, kg*1=kg, mL*1=mL. Also, any quantity *divided* by 1 is also that same initial quantity – e.g., 1/1=1, 2/1=2, mL/1=mL, mg/1=mg, etc.

With that out of the way, it's time to tie all of that review together and do some dimensional analysis. Let's say we have a certain number of milligrams, and we need to figure out how many kilograms our milligrams are equal to. So, we know a quantity with units of mg, and we are trying to convert it to a quantity whose units are kg. Let's begin by looking at the information we *know*, and then defining the information we are trying to *achieve*.

For this example, we will say that we have 123,456 milligrams of something (dopamine, lidocaine, kumquats, whatever), and we are trying to determine how many kilograms of something our initial value is equal to.

Known: 123,456 mg $\Rightarrow \frac{123{,}456mg}{1}$

Goal: ? kg = $\frac{123{,}456mg}{1}$

$$\frac{?\,kg}{1} = \frac{123{,}456mg}{1}$$

We can't solve this problem without additional information. Fortunately, because we covered prefixes before, there is something else we know: there are 1,000 milligrams in a gram, and there are 1,000 grams in a kilogram. Let's state these facts mathematically:

1,000 mg = 1 g

1,000 g = 1 kg

Let's take that first relationship and manipulate it further. If we took that equality and multiplied (or divided) both sides of the equation by some number, would we have the same number? Yes, as long as we multiply (or divide) both sides by the same number, we would have the same quantity. Why would we want to do that? Wouldn't that just be a waste of time? Not if we multiply (or divide) it by something useful. Allow me to demonstrate:

$$\frac{1{,}000mg}{1} = \frac{1g}{1}$$

That's our initial statement, re-written in mathematical form. We can divide anything by 1 and not change its value. Now, I'm the one writing this tutorial, so I choose to multiply both sides of this equation by 1/1,000mg. In reality, I am actually just going to divide both sides of the equation by 1,000mg, since multiplying by 1/1,000mg is the same operation as dividing by 1,000mg:

$$\frac{1{,}000mg}{1} = \frac{1g}{1} \Rightarrow$$

$$\frac{1{,}000mg}{1} \times \frac{1}{1{,}000mg} = \frac{1g}{1} \times \frac{1}{1{,}000mg} \Rightarrow$$

$$\frac{1{,}000mg}{1{,}000mg} = \frac{1g}{1{,}000mg} \Rightarrow$$

$$1 = \frac{1g}{1{,}000mg}$$

Now, what's 1,000mg/1,000mg? Right – it's just 1. So, since 1g/1,000mg is equal to 1, and therefore we can multiply any quantity by 1g/1,000mg and not change that quantity's value, because we simply multiplied it by 1. Shortly we will see why this is useful, but for now just bear with me while I do the same thing for the relationship between grams and kilograms, this time dividing each side by 1,000 grams:

$$\frac{1{,}000g}{1} = \frac{1kg}{1} \Rightarrow$$

$$\frac{1{,}000g}{1} \times \frac{1}{1{,}000g} = \frac{1kg}{1} \times \frac{1}{1{,}000g} \Rightarrow$$

$$\frac{1{,}000g}{1{,}000g} = \frac{1kg}{1{,}000g} \Rightarrow$$

$$1 = \frac{1kg}{1{,}000g}$$

Alright, we've done some seemingly silly math, with no manner of profit in sight. Let's look again at our initial information:

Known: 123,456 mg $\Rightarrow \frac{123{,}456mg}{1}$

Goal: ? kg = $\frac{123{,}456mg}{1}$

$$\frac{?\,kg}{1} = \frac{123{,}456mg}{1}$$

Now let's do something totally crazy (and highly profitable): let's multiply our 123,456 milligrams by one. Heck, let's throw caution to the wind and do it twice. But instead of multiplying 123,456 milligrams by 1, we're going to multiply it by two separate quantities we now know to be equal to 1: 1g/1,000mg, and 1kg/1,000g. Here goes:

$$\frac{123{,}456mg}{1} \times \frac{1g}{1{,}000mg} \times \frac{1kg}{1{,}000g} = ?$$

OK, what did we just do? We took our 123,456 mg, that we're trying to convert to kg, and we multiplied it by 1, and then we multiplied it by 1 again. Did we change our initial value? No – we can multiply our initial value by 1 (or variations thereof) as many times as we like and we will still be left with our initial value. Well then what was the point of all of that? Well, let's finish the equation, and let's rearrange our units and numbers on the right side of the equation:

$$\frac{123{,}456mg}{1} \times \frac{1g}{1{,}000mg} \times \frac{1kg}{1{,}000g} = \frac{123{,}456mg \times 1g \times 1kg}{1 \times 1{,}000mg \times 1{,}000g}$$

Ewww, that's ugly. Let's rearrange it further, still without changing its value:

$$\frac{123{,}456mg \times 1g \times 1kg}{1 \times 1{,}000mg \times 1{,}000g} = \frac{123{,}456 \times 1 \times 1 \times mg \times g \times kg}{1 \times 1{,}000 \times 1{,}000 \times mg \times g}$$

And now let's break it up even further:

$$\frac{123{,}456 \times 1 \times 1 \times mg \times g \times kg}{1 \times 1{,}000 \times 1{,}000 \times mg \times g} = \frac{123{,}456}{1{,}000 \times 1{,}000} \times \frac{mg}{mg} \times \frac{g}{g} \times \frac{kg}{1}$$

Now, what's mg/mg, or g/g? Right – simply 1. Since multiplying by 1 doesn't change anything, let's get rid of those:

$$\frac{123{,}456}{1{,}000 \times 1{,}000} \times \frac{mg}{mg} \times \frac{g}{g} \times \frac{kg}{1} = \frac{123{,}456}{1{,}000 \times 1{,}000} \times \frac{kg}{1} = \frac{123{,}456kg}{1{,}000{,}000}$$

Since we haven't changed anything from our initial value of 123,456 milligrams – because we've done nothing but multiply that value by 1 – let's restate how far we've gone:

$$\frac{123{,}456mg}{1} = \frac{123{,}456kg}{1{,}000{,}000}$$

Have a look at the right side of the equation, specifically the units – the only thing left is kg, or kilograms. Isn't that what we were trying to get to in the first place? Yup. You bet. We've just used dimensional analysis – canceling out those milligrams and grams – to convert our initial milligram value into kilograms. 123,456/1,000,000 works out to be 0.123456 kilograms.

$$\frac{123{,}456kg}{1{,}000{,}000} = 0.123456 \text{ kg}$$

That wasn't so bad, was it? Let's do another, more practical example.

Let's say you have a 180-lb patient. You need to medicate him, but all of the medication dosing regimens call for so much drug per kilogram of body weight. So, we have information with units of lbs., and we need to get it to kg. Let's look again at what we know, and what we are trying to figure out:

Known: 180 lbs. = $\frac{180lbs}{1}$

$$\frac{2.2lbs.}{1kg} = \text{conversion from lbs to kg}$$

Goal: $? \text{ kg} = \frac{180lbs.}{1}$

Now, I skipped a step here. It is common practice to simply write conversion factors as noted above, and now that we've seen how to get there by making a statement and multiplying or dividing both sides, I will abbreviate that from now on. But, the way I wrote that ratio is problematic, which is why we did it that way. Recall that we have lbs. and we're trying to get to kg. See the problem? They way I stated our conversion factor, we have kg on the bottom. So how do we get it on the top? Simple – just flip it.

$$\frac{2.2lbs.}{1kg} \Rightarrow \frac{1kg}{2.2lbs.}$$

So, we've got the kg where we need it, but we still have lbs. in the numerator of our known and another lbs. in the denominator of our conversion factor. But wait a second…if we multiplied those two, wouldn't they cancel and give us the units we are looking for? Yup. You bet:

$$\frac{1kg}{2.2lbs.} \times \frac{180lbs.}{1} = 81.81 \text{ kilograms}$$

And there you have it. This is the essence of dimensional analysis – flipping things, multiplying them out, in order to arrive at our solution, all the while using the units of measure as our guide.

Medical Calculations

Now that we understand mass, weight, volume, and everyone's favorite – dimensional analysis – we turn our attention to the calculations. Though there are thousands of drugs available, it turns out that there are really only four types of calculations, broadly, that we need to know how to do. Now remember, dosing of medications is based on the mass of the drug, but in the end, we need to know how much volume of medication (almost always milliliters, since most meds are dissolved in solution) to administer. So, for each of the 4 calculations, the overall goal is to relate the mass of medication we wish to give to the volume of solution needed to achieve that. This relationship is, of course, dependent on the concentration of drug that we have available. With that said, let's consider each of the four calculations in turn.

Part I: Bolus Medications

1. **The simple mass-based calculation**

   This is the easiest of all of the calculations, and the basis for all of the others. A good example is morphine for adults. Adults typically get morphine in doses of 2 mg at a time. So knowing **how much** medication we want to give, we need to determine how much **volume** in our syringe to push in order to give that much mass of medication. Once we've identified the need for a medication (in this case morphine), and we've checked our 6 rights of medication administration, we take our syringe of medication, and immediately look for the concentration. The concentration, again, is simply a ratio of how much mass of drug is dissolved per unit volume. For morphine, this is typically 10 mg/mL.

   So let's take a look at what information we know, and what information we are trying to achieve. We know that we want to give 2 mg of morphine, and we know that our syringe of morphine contains 10 mg per mL of fluid.

Now what information are we trying to achieve, or in the words of your algebra teacher, "What variable are we trying to solve for?" We are trying to figure out how much volume, or how much of that morphine-laced fluid, to give to the patient. Specifically, we want to know how many mL's of fluid we need to push to give the patient 2 mg of drug.

Now we turn to algebra and dimensional analysis, and we ask ourselves a question: Using the information we **know**, what calculation do we need to perform in order to determine the information we are trying to achieve?

Known: We want to give 2 mg of morphine.

Known: We have morphine 10 mg/mL.

Goal: How many mL should we give?

Since we're using dimensional analysis to solve our problem for us, let's turn our attention to the units of measure. We want to know how many mL's to give. Do we see that unit of measure in any of the information we already know? Yes – it's right there in the concentration. However, there are two things problematic about the concentration: 1) the mL we are looking to solve for is in the denominator, whereas we need it in the numerator; and 2) we have mg (milligrams) in the numerator – we're going to need to get rid of that, by canceling it out somehow. Do we have any known information that will allow us to cancel it out? Yes – we have the mass of the drug that we wanted to give in the first place. Since the piece of information we desire is in mL, and since our known information containing an mL has the mL in the denominator, we need to flip our concentration over, and then multiply:

Known: $\frac{2mg}{1}$ = desired dose $\frac{10mg}{1ml}$ = concentration

Goal: $\frac{?ml}{1}$ of drug to give

First, we flip the concentration, for two reasons: (1) our answer depends on having mL in the numerator; (2) we need a mg in the denominator of the concentration to cancel out the mg in the numerator of the desired dose.

Flip: $\frac{10mg}{1ml} \Rightarrow \frac{1ml}{10mg}$

And now we're ready to multiply:

$\frac{2mg}{1} \times \frac{1mL}{10mg} \Rightarrow \frac{1mL \times 2mg}{1 \times 10mg} \Rightarrow \frac{1mL \times 2}{1 \times 10} \Rightarrow \frac{2mL}{10} \Rightarrow \frac{2}{10}$ mL, which is 0.2 mL.

2. The mass-per-weight calculation

This is pretty much the same as the mass-based calculation, only this time the mass we want to give is a function of the weight of the patient. A good example of a mass-per-weight calculation is pediatric morphine, which is typically given not in 2 mg increments as with adults, but rather as 0.05 – 0.1 mg/kg IV. As you can see, the amount (mass) of drug we want to give is now based on the weight of the patient. In the grand scheme of things, this really only adds one step (multiply the patient's weight by the weight-per-kilo dose), but if we need to convert their weight from English units (pounds, or lbs.) into metric (kilograms, or kg), then technically, it is two extra steps. Let's use a theoretical patient who weighs 66 pounds. We will further assume that he is in a lot of pain, and that we want to give 0.1 mg/kg of morphine, which comes in the same concentration as above (10 mg/mL). First, we will begin again with what we know:

Known:	We want to give 0.1mg/kg of morphine.
Known:	Patient weighs 66 pounds (lbs).
Known:	2.2 pounds (lbs) is equal to 1 kilogram (kg).
Known:	We have morphine 10 mg/mL.
Goal:	Determine patient's weight in kilograms.
Goal:	Determine how much mass (mg) of drug to give.
Goal:	How many mL should we give?

Or, mathematically:

Known: $\frac{0.1mg}{kg}$ = desired dose $\frac{10mg}{1mL}$ = concentration

$\frac{66lbs}{1}$ = patient wt. in lbs $\frac{2.2lbs}{1kg}$ = conversion factor

Goal: $\frac{?kg}{1}$ patient wt. in kg $\frac{?mg}{1}$ of drug to give

$\frac{?mL}{1}$ of drug to give

Let's begin by converting the patient's English weight into metric weight. What do we want to know? We want to know the number of kilograms the patient weighs – so we're looking for a unit of measure that is simply kg, or kg/1. Do we see kg anywhere in the information that we know? Yes – it's right there in the pounds-to-kilograms conversion factor, whose units are lbs/kg. Since it's in the denominator, we're going to need to flip it, which will give us units of kg/lbs. Next, we see that with our flipped units, we have a lbs unit in the denominator that we need to get rid of. How can we do that? Simple – by multiplying our new, flipped conversion factor by the patient's weight in pounds:

Flip: $\frac{2.2lbs}{1kg} \Rightarrow \frac{1kg}{2.2lbs}$

Calculate: $\frac{44lbs}{1} \times \frac{1kg}{2.2lbs} \Rightarrow \frac{1kg \times 66lbs}{1 \times 2.2lbs} \Rightarrow \frac{1 \times 66kg}{1 \times 2.2} \Rightarrow \frac{66kg}{2.2} \Rightarrow \frac{66}{2.2}$ kg,

which works out to be 30 kilograms.

We're still trying to figure out how many mL's of morphine to give to our 66-lb. child who is in a lot of pain. We have just created new information, so let's review what we know now that we've done some work:

Known: $\frac{0.1mg}{kg}$ = desired dose $\frac{10mg}{1mL}$ = concentration

$\frac{30kg}{1}$ = patient wt. in kilograms

Goal: $\frac{?mg}{1}$ of morphine to give $\frac{?mL}{1}$ of morphine to give

Now that we have the patient's metric weight, we turn our attention to trying to figure out how much mass (i.e., how many mg) of drug to give the patient. We see the mg we're looking for in our desired dose, but it's got that pesky kg in the denominator. Can we get rid of that kg somehow? Sure! All we have to do is multiply the desired dose by the patient's weight in kilograms, which is what we just calculated:

$\frac{0.1mg}{1kg} \times \frac{30kg}{1} \Rightarrow \frac{0.1mg \times 30kg}{1 \times 1kg} \Rightarrow \frac{0.1 \times 30mg}{1} \Rightarrow \frac{3mg}{1} \Rightarrow \frac{3}{1}$ mg, or just 3 mg.

At this point, our calculation is exactly the same as the simple mass-based calculation that we did in Part 1. We know how much drug we want to give, and we know our concentration. Again, at this point we want to know how many mL's of drug to push, and seeing the mL in the denominator of our concentration, we again flip the concentration in order to get the mL's in the numerator. But before we do that, just for the sake of completeness, let's review the things we know right now:

Known: $\frac{3mg}{1}$ = desired dose $\frac{10mg}{1mL}$ = concentration

Goal: $\frac{?\,mL}{1}$ of morphine to give

Flip: $\frac{10mg}{1mL} \Rightarrow \frac{1mL}{10mg}$

Once again, it's time to cancel some units and do some multiplication to arrive at that piece of information we desire:

$$\frac{3mg}{1} \times \frac{1mL}{10mg} \Rightarrow \frac{1mL \times 3mg}{1 \times 10mg} \Rightarrow \frac{1mL \times 3}{1 \times 10} \Rightarrow \frac{3mL}{10} \Rightarrow \frac{3}{10}$$ mL, which is 0.3 mL.

Alright then. At this point in our mathematical adventure, we're halfway done. The above information is really all you need in order to give a bolus of any medication to any patient. That's the good news. The bad news, of course, is that the only other thing for us to go over is…drips! Drips aren't really any harder, they just require a few more steps, and thus, a few more calculations.

Part II: Drip Medications

With medication drips things start to get a little bit hairy. We have several new variables to deal with. First, drips are usually specified in minutes (though some drips are specified in hours), so we have to deal with time. Second -- and this is why prehospital med math is harder than in-hospital med math – we have to take into account what drip set we are using. In the hospital it doesn't matter – they are using pumps, and the pumps don't care about the drip set – all the pump needs to know is how much volume to give over how much time. We are concerned with this variable prehospitally as well, but since we don't always have pumps in the prehospital arena, the way in which we work to deliver a specified volume of medication over a specified period of time is by calculating, and then adjusting our roller clamp to deliver, a certain number of drops per minute. Of course, since different drip sets deliver a different volume of fluid per drip, we must base our calculations on the drip set we are using. The number of drops per minute we give is the ultimate goal of prehospital drip calculations, and it requires a fair amount of work to get there.

Before we begin, let's go over some important and rather practical information. First off, using a roller-clamp and a stopwatch to calculate a drip rate is considered a rude, crude, and lewd method of dripping meds into a patient; but for some systems, particularly volunteer-based systems, it's the only option because of the high cost of pumps. With that said, if your organization does interfacility runs or simply transports patients on drips with any regularity, you might consider that the cost of some pumps is going to be less than any litigation resulting from a lawsuit filed because of inappropriate drip-medication administration.

Having said that, we will be doing our drip calculations based on the assumption that you are being crude and titrating drip rate with a roller clamp. The nice thing about doing it this way is that if the pump fails or otherwise malfunctions, you can still run the drip. What's more, performing the calculations this way will also show you how to set up a pump if you have one. The nicer pumps, usually found in-hospital, will do **all** of the calculations for you – you simply tell the pump the patients weight, the volume of the bag, the amount of drug you put in the bag, and how much medication you want to give per unit time, and the pump does the rest – some will even convert all of the necessary units for you. The lesser pumps – and usually the ones, if any, found in ambulances and helicopters – require you to do most of the calculations, and then tell the pump how much fluid to push over how much time. Part of calculating all the way down to drops per minute involves calculating the number of mL's per minute, and this is usually the variable you will need to input to a prehospital IV pump.

The last thing to cover involves drip sets themselves. Drip set tubing comes in a wide range of specifications. The old standards were 10-gtt and 60-gtt drop sets, o the gtt-number indicating the number of drops for that set required to deliver 1 mL of fluid. Recently we have seen growing popularity of 15- and 20-gtt drop sets. The most accurate drip set to use is the 60-gtt drop set, because the divisions per cc are finer, and thus more accurate. There is one caveat to this, however: eventually, as you increase the flow rate, you will no longer be able to count the number of drops being delivered per minute, as the flow from the drop set will eventually become a steady stream of fluid. For a 60-gtt drop set, this limit is in the range of 120-130 drops per minute; there are similar limits for the other drop sets. But for a 60-gtt drop set, this means that the maximum volume of fluid we can accurately deliver per minute is just about 2 cc's, since 120 drops from a 60-drop set is 2 cc's of fluid. This limitation of drip set tubing is yet another factor that we must take into account when we mix our medication.

3. Mass-per-time drips

Of the drips, this is of course the easiest. The overall goal is to deliver a certain amount (here, mass, usually mg or mcg) of drug over a certain period of time (almost always minutes, but occasionally, hours). A good example would be a lidocaine or amiodarone drip, but I'm going to be different and use neosynephrine as our example, because it will be more illustrative.

The first question we must ask ourselves once we've decided to do any drip is: How should we mix it? We have several options when mixing up the drip – first, what size bags of fluid do we have available – usually drips are done with 50, 100, 250, or 500 cc bags. Second, how much medication should we put in the bag? There are, fortunately, two factors which will do most of the decision making for us. The first, and most important factor, is the maximum rate at which the drip will run. For example, neosynephrine should never be delivered at any rate higher than 300 mcg per minute. As we discussed above, using a 60-gtt drop set will afford us the most accurate delivery of medication, so for this example, we will use that. How fast can we run it? Up to 120 drops per minute, or 2 mL's per minute. So, under ideal circumstances, we should mix our neosynephrine so that 120 drops per minute delivers 300 mcg per minute. Let's state this mathematically:

Known:	Max. rate of neosynephrine is 300 mcg/minute.
Known:	We selected a 60-gtt drop set, so our maximum rate is 120 gtts/minute.
Goal:	Determine the ideal concentration to mix.
Goal:	Determine the minimum flow rate in mL/min.
Goal:	Determine the minimum flow rate in gtts/min.

Goal: Determine the maximum flow rate in mL/min.

Goal: Determine the maximum flow rate in gtts/min.

Goal: Determine a specified flow rate in mL/min.

Goal: Determine a specified flow rate in gtts/min.

The goal here is somewhat misleading, particularly in the prehospital arena. Mixing is ideally dependent on the above knowns, but sometimes it is practically limited by the equipment you have on hand. For instance, the squad I volunteer for stocks nothing but 1000 cc bags of saline, so 50, 100, 250, or even 500 bags of fluid are simply not an option. Secondly, we have to take into consideration the amount of drug we have on hand. Realistically, because there are so many variables to play with here, for every drug you are allowed to administer, these types of calculations should be carried out beforehand (i.e., when the protocol is written), and there should be standard mixing guidelines for every drug in your arsenal. This saves us a lot of calculation in the field, which is a good thing. For instance, the industry-standard mix of dopamine is to put 400 mg in 250 mL of fluid, yielding a concentration of 1600 mcg/mL. Having a standard concentration is very helpful, as we will see. The other more practical part of this is that 2 mL/minute works out to be 120 mL/hour. This is something else we must pay attention to, since we don't want our drip to have the side effect of volume-overloading the patient. If our flow rate worked out to be in the liters per hour, we would have to worry about that as well.

But let's continue with the example. Let's just assume we want to use a 250-mL bag. This is a good choice, since a 250-mL bag at the max rate of 2 mL/minute will last 2 hours and 5 minutes, enough for us to get the patient to the hospital, and allow the ER staff to stabilize the patient before they have to worry with mixing up another bag. It's also a good size bag because 2 hours covers almost all interfacility transport times – probably better than 95% of them. In addition, as noted above, the maximum flow rate we chose will not volume-overload the patient. So we will add this to our list of information:

Known: Max. rate of neosynephrine is 300 mcg/minute.
Known: We selected a 60-gtt drop set, so our maximum rate is 120 gtts/minute, which is 2 mL/minute.
Known: We will use a 250 mL bag.
Goal: Determine the ideal concentration to mix.

Now since we know what size bag we're going to use, we can identify a more specific goal: how much medication to put in the bag. That goal depends on the "ideal" concentration, so we need to determine that first. Since the maximum rate of neosynephrine is 300 mcg/minute, we want that rate to coincide with a maximum flow rate of 2 mL per minute (again, this will be the most accurate refinement). What we are looking for, then, is how many micrograms per mL of fluid do we need:

Known: $\frac{300mcg}{1\min}$ = maximum rate $\frac{2mL}{1\min}$ = maximum flow rate

Goal: $\frac{?mcg}{1mL}$ for our concentration

We've already stated the piece of information we're looking for: how many micrograms per milliliter to mix. Do we see micrograms in the above knowns? Yes – it's right there in the maximum rate, and it's right where it needs to be, in the numerator. Do we see mL in the knowns? Yes – in the numerator, but we need it in the denominator. What do we need to do? That's right, flip it. And since both of the units in both of the knowns above also contain min, and since we're flipping one over, those will simple cancel out:

Flip: $\frac{2mL}{1\min} \Rightarrow \frac{1\min}{2mL}$

Calculate:

$$\frac{300mcg}{1\min} \times \frac{1\min}{2mL} \Rightarrow \frac{300mcg \times 1\min}{1\min \times 2mL} \Rightarrow \frac{300mcg \times 1\min}{2mL \times 1\min} \Rightarrow \frac{300mcg}{2mL} \Rightarrow \frac{150mcg}{1mL}$$

With that information added to our known database, let's take another look at what we know, and what we want to know:

Known: We need a concentration of 150 mcg/mL

Known: We will use a 250 mL bag.

Goal: How much mass of drug do we need to put in the bag to get 150 mcg/mL?

Mathematically:

Known: $\frac{150mcg}{1mL}$ = desired concentration $\frac{250mL}{1}$ = bag size

Goal: $\frac{?mcg}{1}$ do we need to create the desired concentration?

So we're looking for an answer in mcg. Do we see that anywhere in the information that we know? Yes – it's in the concentration, and it's in the numerator, where we need it. You should be able to see by now that the mL in the denominator of the concentration will be canceled by numerator of the bag size:

$$\frac{150mcg}{1mL} \times \frac{250mL}{1} \Rightarrow \frac{150mcg \times 250mL}{1 \times 1mL} \Rightarrow \frac{150 \times 250mcg}{1} \Rightarrow \frac{37500mcg}{1}$$

So we know we need 37,500 micrograms of neosynephrine to create our perfect recipe. Problem is, the vials of neosynephrine don't – and won't – tell you how much drug is in them. In reality, the do tell you, they just do it in a subtle way: neosynephrine 1%. This presents us with 2 problems.

1. Our desired mass in micrograms, not %.
2. The easiest way to go from % to mass is to multiply the percent number by 10, which will yield the number of *milligrams* per mL.

   So, first we need to convert out 37,500 micrograms to milligrams, then convert our % to milligrams, and then figure out how many mL's of neosynephrine solution we need to inject into our bag to arrive at our ideal concentration.

1. Micrograms to milligrams

   Known: $\frac{37500mcg}{1}$ = what we want $\frac{1000mcg}{1mg}$ = mcg to mg

   Goal: $\frac{?mg}{1}$ do we need?

   Again, you should be able to see by now that 1) we need to flip our conversion factor and 2) from there units will cancel and give us what we want:

   Flip: $\frac{1000mcg}{1mg} \Rightarrow \frac{1mg}{1000mcg}$

   And now, calculate:

$$\frac{37500mcg}{1} \times \frac{1mg}{1000mcg} \Rightarrow \frac{1mg \times 37500mcg}{1 \times 1000mcg} \Rightarrow \frac{37500mg}{1000} \Rightarrow \frac{37.5mg}{1}$$

So, we're going to need 37.5 mg of drug for our recipe. Next up, convert % solution to milligrams:

2. % Solution to Milligrams

Known: $\frac{1\%}{1mL}$ = solution strength $\frac{10mg}{1\%}$ = mg to % conversion

Goal: $\frac{?mg}{1mL}$ is in the 1% solution

There is no need to flip anything here, just multiply:

$$\frac{1\%}{1mL} \times \frac{10mg}{1\%} \Rightarrow \frac{10mg \times 1\%}{1mL \times 1\%} \Rightarrow \frac{10mg}{1mL} \Rightarrow \text{10 mg/mL solution}$$

Let's review what we know right now:

Known: We need 37.5 mg of neosynephrine for our recipe.

Known: Neosynephrine comes 10 mg/mL

Goal: How many mL's of neosynephrine do we need to create our perfect little concoction?

Known: $\frac{37.5mg}{1}$ = needed $\frac{10mg}{1mL}$ = concentration

Ready for the math? Again, since we're looking for something that's in mL, and since what we know contains mL in the denominator, we flip it. As a bonus, when we go to multiply it out, having flipped it, our mg's will cancel out:

Flip: $\frac{10mg}{1mL} \Rightarrow \frac{1mL}{10mg}$

And now, calculate:

$$\frac{37.5mg}{1} \times \frac{1mL}{10mg} \Rightarrow \frac{1ml \times 37.5mg}{1 \times 10mg} \Rightarrow \frac{37.5mL}{10} \Rightarrow \frac{3.75mL}{1} \Rightarrow 3.75mL$$

So, in order to have our neosynephrine mixed at that ideal 150 mcg/mL, we would need to put 3.75 mL's of 1% neosynephrine solution into our 250 mL bag. For those of you out there who like to pay attention to the exceptionally fine details, yes, this would yield a total final volume of 253.75 mL in the bag. Usually, when the volume of injectate is small compared to the total volume of the bag, this extra volume is ignored; however, if you want to be totally precise, you could just as easily remove 3.75 mL of fluid from the bag prior, and only prior, to injecting the Neosynephrine into the bag.

(Note: We've already done quite a bit of calculation, and we really haven't even started yet. The above calculations were just a taste of something you might have to do yourself either in the field when drip charts don't apply or you don't have software; it's also a good example of the thought process that goes into creating a standard mixing protocol for a new drug. Normally, this is known beforehand, and typically a mass-based calculation proceeds at the starting point below.)

So now that we've mixed our neosynephrine, and mixed it perfectly, what should we do? Let's say the range for neosynephrine is 10-300 mcg/min. Unless you have a specific reason for starting somewhere else with a drip (and when we get to dopamine, we'll see that sometimes we do), it's always wise to start at the bottom. So we'll start with 10 mcg/min:

Known: We want to start at 10 mcg/min.

Known: Our neosynephrine is 150 mcg/mL.

Goal: How many mL/min gives us 10 mcg/min?

Goal: How many gtts/min gives the mL/min that we just calculated?

Or, mathematically:

Known: $\frac{10mcg}{1\min}$ = start rate $\frac{150mcg}{1mL}$ = concentration

Goal: $\frac{?\,mL}{\min}$ = $\frac{?\,gtts}{\min}$ = $\frac{10mcg}{\min}$

And once again, since we're looking for mL/min, we flip, multiply, and cancel as necessary:

Flip: $\frac{150mcg}{1mL} \Rightarrow \frac{1mL}{150mcg}$

$$\frac{10mcg}{1\min} \times \frac{1mL}{150mcg} \Rightarrow \frac{1ml \times 10mcg}{1\min \times 150mcg} \Rightarrow \frac{10mL}{150\min} \Rightarrow \frac{1mL}{15\min} \Rightarrow 0.067mL/\min$$

Now let me digress again with some practical information. We already determined the maximum drip rate when we did the mixing – it's 2 mL/min, or with our 60-gtt drop set, 120 gtts/minute, or I'll even accept, "slow enough that we can count it," since 120 is about the maximum countable rate for a 60-gtt drop set. In addition to being a good place to start, the rate we just calculated is also something else – it's the minimum flow rate, unless we want to turn that particular med off completely. So, once we figure out how many drops per minute 0.067mL/minute works out to be, with our 60-gtt drop set, we'll have both the maximum and minimum drip rates. And if we want to be slackers, we can run our neosynephrine drip anywhere in between those two values and "titrate to effect." So without further ado, let's figure out that minimum drip rate:

Known: $\frac{0.067 mL}{1 \min}$ = minimum rate $\frac{60 gtts}{1 mL}$ = drop set

Goal: $\frac{? gtts}{\min} = \frac{0.067 mL}{\min}$

And since we're looking for gtts/min, we don't even have to flip anything – direct multiplication will cancel the mL's and leave us with our 'units of desire:'

$$\frac{0.067 mL}{1 \min} \times \frac{60 gtts}{1 mL} \Rightarrow \frac{60 gtts \times 0.067 mL}{1 \min \times 1 mL} \Rightarrow \frac{4 gtts}{1 \min} \Rightarrow 4 \textit{ gtts/min}$$

And there you have it – 4 drops per minute, or one drop every 15 seconds, is our minimum rate; further, we already know and have discussed that, because of our super-savvy mixing, our maximum rate is 120 drops per minute. We can run it anywhere from 4-120 drops per minute, monitor vitals, and adjust the rate based on changes in vitals and the known effects of the drug. Even more practical, 4 gtts/min is 10 mcg/min. We can go all the way up to 300 mcg/min, and when trying to normalize someone on neosynephrine, it turns out that moving up in 10 mcg/min increments isn't a bad way to do it. So we simply increase our drip rate 4 gtts/minute at a time, reassess, and further titrate as necessary (4 gtts/min, then 8 gtts/min, then 12, 16, 20, and so on, all the way up to that 120 we've already discussed).

But what happens when we call the hospital, speak to the doc, and he wants us to run it at a certain rate?

Fortunately, we don't have to start all over again. Because we mixed it wisely, it's impossible for him to give us a rate that we can't do – we can go all the way up to the maximum rate, and we'll still be able to count it. As far as achieving a specific rate that a doc gives us, we again call attention to what we know (and we'll use 73 mcg/min as a random rate that a doc might specify):

Known: $\frac{73mcg}{1\min}$ = MD request $\frac{150mcg}{1mL}$ = concentration

Goal: $\frac{?\,mL}{\min} = \frac{73mcg}{\min}$

We're looking, once again, for mL/min, so we flip, multiply, and cancel as necessary:

Flip: $\frac{150mcg}{1mL} \Rightarrow \frac{1mL}{150mcg}$

$$\frac{73mcg}{1\min} \times \frac{1mL}{150mcg} \Rightarrow \frac{1mL \times 73mcg}{1\min \times 150mcg} \Rightarrow \frac{73mL}{150\min} \Rightarrow 0.487mL/\min$$

And we review what we now know:

Known: $\frac{0.487mL}{1\min}$ = minimum rate $\frac{60gtts}{1mL}$ = drop set

Goal: $\frac{?\,gtts}{\min} = \frac{0.487mL}{1\min}$

It's essentially the same calculation as before, with the same cancellation:

$$\frac{0.487mL}{1\min} \times \frac{60gtts}{1mL} \Rightarrow \frac{60gtts \times 0.487mL}{1\min \times 1mL} \Rightarrow \frac{29.2gtts}{1\min} \Rightarrow 29.2 \; gtts/min$$

In reality, this is impossible. The doc will have to settle for 29, or even 30 gtts/min, if we don't have a pump; this is a pretty good example of why you need a pump to do drips with any accuracy (and in fact, you'd be hard pressed to find an organization or institution that will allow clinicians to run neosynephrine without a pump).

Let's do another example, this time with a very common medication: amiodarone. Amiodarone can be given for stable ventricular tachycardia as 150 mg over 10 minutes, or post-resuscitation, 1 mg/min for the first 6 hours and then 0.5 mg/min for the remaining 18 hours (and not to exceed a total dose of 2.2 g of amiodarone within 24 hours).

For the 10-minute drip, most institutions use a 100 mL bag of D_5W, though it is perfectly acceptable to use 0.9% normal saline.[2,3] We will of course be using standard amiodarone, which is supplied in 3 mL vials containing 150 mg of drug, for a total concentration of 50 mg/mL. Now since 150 mg is in a vial and 150 mg is what we want to give over 10 minutes, we will

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[2] This is controversial, and usually poorly understood by those stirring up the controversy. Many medications, among them nitroglycerin and amiodarone will, if left in solution for long durations, bind to the plastic bags and tubing. When this happens, you are giving at best a reduced concentration of medicine, and at worst a saline drip. This is why nitroglycerin and amiodarone have to be kept in glass bottles. However, for amiodarone, this binding to plastic does not occur for at least two hours, so we can easily use it for a 10-minute drip and not worry about it.

[3] The other controversial issue with critical care drips is what fluid to use. For amiodarone, D_5W is the best, due to the chemical nature of amiodarone, because amiodarone will dissolve most equally in D_5W. If you have D_5W available, use it; if saline is all you have, amiodarone will dissolve just fine in that as well.

simply put 3 mL of our amiodarone into our 100 mL bag, but we will of course first remove 3 mL of fluid from our bag, so we don't end up with a bag containing 103 mL's of fluid.

So, we've mixed up our amiodarone and we now have 150 mg in 100 mL, or just 1.5 mg/mL. We could do some fancy math to arrive at a rate of 15 mg/min, but it is easier to realize that since we have mixed our mass in 100 mL of fluid, it is now the 100 mL of fluid we need to give over 10 minutes:

$$\frac{100mL}{10\min} = \frac{10mL}{1\min}$$

Now stare at that for a second. How fast can we run a 60-gtt set? Not much more than 120 gtts/min, or 2 mL/min. We're way above that, so we will need to use a 10-gtt set. Here comes the drip rate:

$$\frac{10mL}{1\min} \times \frac{10gtts}{1mL} = 100 \text{ gtts/min.}$$

4. Mass-per-weight-per-time drips

This is the last type of calculation to master. As with the boluses, once we've factored in the patient's weight, it simply reduces to a simple mass-per-time drip. As an example medication, we'd be hard pressed not to use dopamine, since it is the most common drip administered by prehospital providers. Also, since the above discussion mentioned that dopamine is almost always mixed to a standard 1600 mcg/mL, it allows us to gloss over the discussion of mixing the ideal concentration, which again, is something that should be determined before the providers get their hands on the drug (i.e., it should be written into the protocol before the protocol is rolled out). Anyways, knowing the concentration, let's get started.

There are several ways to arrive at standard-concentration dopamine. The most common method is to inject 400 mg of drug into 250 mL of fluid, which yields the standard concentration of 1600 mcg/mL, as we will see shortly. You can also put 800 mg into 500 mL, or even 1600 mg into 1000 mL. The end result is the same:

$$\frac{400mg}{250mL} \times \frac{1000mcg}{1mg} \Rightarrow \frac{400{,}000mcg}{250mL} \Rightarrow \frac{40{,}000mcg}{25mL} \Rightarrow \frac{1600mcg}{1mL}$$

For this example, we will assume we have a patient who weighs 176 lbs. Dopamine is administered at 5-20 mcg/kg/minute. A good place to start is 5 mcg/kg/minute, so let's begin the thought process:

Known:	Patient weighs 176 lbs
Known:	Dopamine is given 5-20 mcg/kg/minute, we will start a 5 mcg/kg/minute.
Known:	Our dopamine is mixed to 1600 mcg/mL.
Goal:	Convert patient's weight in lbs to kg.
Goal:	Make sure that at the maximum drip rate, we will be able to count it accurately.
Goal:	How many mL/minute do we need for 5 mcg/kg/min?
Goal:	How many gtts/minute do we need for 5 mcg/kg/min?
Goal:	How many mL/minute do we need for 20 mcg/kg/min?
Goal:	How many gtts/minute do we need for 20 mcg/kg/min?

So let's begin the fun.

Known: $\frac{176lbs}{1}$ = pt. weight, lbs $\frac{1kg}{2.2lbs}$ = kg to lbs

Goal: $\frac{?\,kg}{1}$

The math should be fairly obvious by now. Here goes:

$$\frac{176lbs}{1} \times \frac{1kg}{2.2lbs} \Rightarrow \frac{1kg \times 176lbs}{1 \times 2.2lbs} \Rightarrow \frac{176kg}{2.2} \Rightarrow 80kg$$

Now ultimately we want to know how many drops per minute to give. To calculate that, we need to know how many milliliters per minute to give. To calculate that, we need to know the concentration, and we need to reduce the weight-based dosing range to a simple mass-based dosing range:

5 mcg/kg/min to ? mcg/min

Known: $\frac{5mcg}{kg/\min}$ = current dose $\frac{80kg}{1}$ = pt. weight

Goal: $\frac{?\,mcg}{1\min}$ = reduced to mass-based dose

And we can see that simple multiplication will cancel the patient's weight:

$$\frac{5mcg}{kg/\min} \times \frac{80kg}{1} \Rightarrow \frac{5mcg \times 80kg}{1 \times kg/\min} \Rightarrow \frac{400mcg}{\min}$$

So, we now know that for a 176-lb patient, 5 mcg/kg/min works out to be 400 mcg/min.

Now, we could save a lot of recalculation by realizing that the 20 mcg/kg/minute maximum is simply 4 times the 5 mcg/kg/minute minimum, and simply multiply the 400 mcg/min dose by 4 to arrive at a maximum mass rate of 1600 mcg/minute. But the point of this paper is to illustrate the mathematics, so here we go:

20 mcg/kg/min to ? mcg/min

Known: $\frac{20mcg}{kg/\min}$ = current dose $\frac{80kg}{1}$ = pt. weight

Goal: $\frac{?mcg}{1\min}$ = reduced to mass-based dose

And we can see that simple multiplication will cancel the patient's weight:

$$\frac{20mcg}{kg/\min}\times\frac{80kg}{1}\Rightarrow\frac{20mcg\times 80kg}{1\times kg/\min}\Rightarrow\frac{1600mcg}{\min}$$

Having calculated out the maximum mass rate, we now need to look at the volume rate required to achieve that, and make sure that it will be countable with the 60-drop set that we will be using. Here goes:

Known: $\frac{1600mcg}{\min}$ = max. rate $\frac{1600mcg}{1mL}$ = concentration

Known: $\frac{60gtts}{mL}$ = our drop set

Goal: $\frac{?mL}{1\min}$ = maximum volume

Goal: $\frac{?gtts}{1\min}$ = maximum drip rate

Once again, we find ourselves looking for an answer whose units are mL/min, and then an answer whose units are gtts/min. Turning to the former goal, you should be able to see by now that we can get the mL/min that we are looking for by flipping the concentration, and then multiplying that by the maximum rate:

Flip: $\frac{1600mcg}{1mL} \Rightarrow \frac{1mL}{1600mcg}$

$$\frac{1600mcg}{1\min} \times \frac{1mL}{1600mcg} \Rightarrow \frac{1mL \times 1600mcg}{1\min \times 1600mcg} \Rightarrow \frac{1mL}{\min} \Rightarrow 1mL/\min$$

Now, it should be commonsensical that 1mL/min with a 60-gtt/mL drop set will simply be 60 gtts/min, but let's do the math anyways:

Known: $\frac{1mL}{\min}$ = starting rate

Known: $\frac{60gtts}{mL}$ = our drop set

Goal: $\frac{?\,gtts}{1\min}$ = starting drip rate

And here we go:

$$\frac{1mL}{1\min} \times \frac{60gtts}{1mL} \Rightarrow \frac{60gtts \times 1mL}{1\min \times 1mL} \Rightarrow \frac{60gtts}{1\min}$$

And we should also recall from our prior discussion that with a 60-gtt set, we can go all the way up to 2 mL/min before we lose the ability to count it, so we can actually go up to twice the maximum rate before we lose count.

Knowing that our mixture is feasible, it's time to return to the more immediate goal: starting the patient at 5 mcg/kg/min. Again, we could look at the relationship between 5 mcg/kg/min and 20 mcg/kg/min (a simple factor of 4), and then relate that to our already-calculated rate of 1mL/min for 20 mcg/kg/min, but this is a paper about medical mathematics, so we're just going to do it:

Known: $\frac{400mcg}{\min}$ = starting rate $\frac{1600mcg}{1mL}$ = concentration

Known: $\frac{60gtts}{mL}$ = our drop set

Goal: $\frac{?\,mL}{1\min}$ = starting volume

Goal: $\frac{?\,gtts}{1\min}$ = starting drip rate

Once again:

$$\frac{400mcg}{1\min} \times \frac{1mL}{1600mcg} \Rightarrow \frac{1mL \times 400mcg}{1\min \times 1600mcg} \Rightarrow \frac{1mL}{4\min} \Rightarrow \frac{0.25mL}{\min} \Rightarrow 0.25mL/\min$$

We add this volume per minute to our known information:

Known: $\frac{1mL}{\min}$ = starting rate

Known: $\frac{60gtts}{mL}$ = our drop set

Goal: $\frac{?\,gtts}{1\min}$ = starting drip rate

And proceed to get our drip rate:

$$\frac{0.25mL}{1\min}\times\frac{60gtts}{1mL}\Rightarrow\frac{60gtts\times 0.25mL}{1\min\times 1mL}\Rightarrow\frac{15gtts}{1\min}$$

And we're done.

Finishing Up

Alright, we've covered the 4 types of calculations – a breeze, right? There's just a few other issues to discuss, and then I'll do a few nonstandard calculations that will hopefully get you into the idea of generalizing what we've done. In parting I'd like to stress that the only way to learn medical mathematics (or *any* mathematics, for the matter) is by *doing* medical mathematics. A lot of it.

Funky Units

Some of these I've already glossed over; others I haven't even discussed. Let's get into the details.

Grains

You might go your entire career and never encounter the grain as a unit of measure, but if you do see it, you will probably have a very difficult time figuring out what it is. The grain, quite simply, is a measure of mass, where 1 grain = 60 mg. You'll typically see it in apothecaries, herbal prescriptions, etc. Other than that, no one really uses it anymore.

Epinephrine 1:1,000 and 1:10,000

Another funky unit of measure; 1:1,000 and 1:10,000 simply tell you the concentration of epinephrine, but in a totally non-intuitive way. Water has a density of 1 gram per mL, meaning that 1 mL of water weighs exactly 1 gram. Converting to milligrams, 1 mL of water also weighs 1,000 mg. So if you've got 1 mg of epinephrine dissolved in 1 mL of water, what you have is a mass-based ratio of 1 mg epinephrine to 1,000 mg of water (i.e., 1:1,000). Same thing goes for epinephrine 1:10,000 – 10 mL of water is 10,000 mg of water, with 1 mg of epinephrine dissolved in it.

% Solutions

Similar to the mass-based ratios for epinephrine discussed above, here we have another non-standard, non-intuitive method of informing you of the concentration. The % strength concentration is calculated by multiplying the number of grams per mL by 100:

$$\frac{\#g}{mL} \times 100 = \%$$

I'll just save some time here. 1% solutions contain 10 mg of drug per mL:

$$\frac{0.010g}{mL} \times 100 = 1\%$$

And you should be able to determine by now that 10mg = 0.010g. So, we could deal with grams to get to the concentration; but, since most meds are thought of in milligrams, the better shortcut is to simply take the % number and multiply it by 10, to arrive at the concentration in mg/mL. So 1% propofol is 10 mg/mL of propofol, 4% lidocaine contains 40 mg/mL of lidocaine, 50% dextrose contains 500 mg/mL of sugar, and 10% calcium chloride ($CaCl_2$) contains 100 mg/mL of calcium.

Other Calculations

Let's do a few non-standard, but not uncommon, calculations: heparin bolus and drip, oral prednisone, and even a Parkland-formula calculation.

Heparin Both Ways

First the bolus, then the drip. Heparin is first bolused at 60 IU/kg (maximum bolus: 4000 IU), then run as a drip at 12 IU/hr (max. infusion rate: 1,000 IU/hr), where the drip is rounded to the nearest 50 IU/hr.

So your first thought might be (and hopefully is, since that's the only reason I'm running through this): what happened to mg, and what are IU? IU is the abbreviation for international units. You see, heparin is not a single molecule – rather, it's a string, or chain, of a smaller molecule, or monomer, strung or chained together. Each polymeric chain of heparin has a different length, and therefore a different weight, so it is impossible to dose 'normal' heparin by weight.[4] Since we can't dose heparin by weight, we have to dose it by something else, and something else that is relevant to the drug and its effects. 1 IU of heparin is the amount of heparin necessary to keep 1 mL of sheep's blood from clotting for 1 hour. Past that, it doesn't really matter – we know how many IU of heparin we have per mL, and we treat the mathematics the same. Heparin for injection is packaged as 1,000 IU in 2 mL of diluent; heparin drips are standardized at 25,000 IU in 250 mL of fluid.

For this scenario, we will use a patient who weighs 200 lbs. Without further ado, the bolus calculation:

Known: We need to give 60 IU/kg.

Known: Patient is 200 lbs.

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[4] Lovenox makes this statement completely untrue. Lovenox, or enoxaparin, is a so-called "low-molecular weight heparin" – it has been purified so that only heparin chains of a specific length, and thus a specific weight, are allowed to become enoxaparin. Because every heparin molecule in enoxaparin has the same molecular weight, enoxaparin can, and is, dosed in mg/kg.

Known: 2.2 lbs is 1 kg.

Goal: We need to determine how many kilos he is.

Goal: We need to figure out how many IU total to give.

Goal: We need to figure out how many mL to give.

1. Get that weight to kilos.

$$\frac{200lbs}{1} \times \frac{1kg}{2.2lbs}$$ = 91 kg (90.9, close enough)

2. How many IU's are we giving?

$$\frac{91kg}{1} \times \frac{60IU}{kg}$$ = 5460 IU

But wait, there's a problem. The max. bolus for heparin is 4,000 IU, and since we're above that, he is simply going to get 4,000 IU.

3. How many mL's are we giving?

$$\frac{4000IU}{1} \times \frac{1mL}{500IU}$$ = 8mL

Above we've stated that we have Heparin for injection, which is 1,000 IU in 2 mL. Proving that those numbers work out to 500 IU/mL is an exercise I'll leave to you.

OK, time for his drip. Again, it's 12 IU/hr (not per min), and we have on hand a bag with 25,000 IU in 250 mL. That's 1,000 IU per mL, which again I leave up to you to calculate. Here goes:

Known: We need to give 12 IU/kg/hr.

Known: Patient is 91kg.

Known: We have a bag with 1,000 IU per mL.

Known: There are 60 minutes in 1 hour.

Goal: We need to determine how many IU per hour he gets.

Goal: We need to figure out how many mL's per hour that is.

Goal: We need to figure out how many gtts per minute that is.

1. How many IU per hour?

$$\frac{91kg}{1} \times \frac{12IU}{1kg/hr} = 1{,}092 \text{ IU/hr.}$$

And again he is above the maximum of 1,000 IU per hour. So we need to give 1,000 IU per hour, and while I think all of you can see that that's just 1 mL per hour, let's just do it anyway:

2. How many mL's per hour?

$$\frac{1{,}000IU}{1hr} \times \frac{1mL}{1000IU} = 1 \text{ mL/hr}$$

Lastly, and I hope you all see the benefit of using a 60-gtt set here, the drip rate:

3. How many drops per minute?

$$\frac{1mL}{1hr} \times \frac{1hr}{60\min} \times \frac{60gtt}{1mL} \Rightarrow \frac{1mL \times 1hr \times 60gtt}{1mL \times 1hr \times 60\min} = 1 \text{ gtt/min.}$$

Oral Prednisone

Next up, corticosteroids for asthma. IV we have methylprednisolone, or Solu-Medrol, and orally we have Prednisone tablets, 20 mg per tablet. Typical dosing for asthma is 1 mg/kg, and we will use a prototypical 154-lbs. patient.

Known:	Patient is 154 lbs.
Known:	We want to give 1 mg/kg prednisone.
Known:	We have prednisone 20 mg/tablet.
Goal:	How many kilos is he?
Goal:	What's the total dose?
Goal:	How many tablets is that?

1. How many kilos is he?

$$\frac{154 lbs}{1} \times \frac{1 kg}{2.2 lbs} = 70 \text{ kg}$$

2. What's the total dose?

$$\frac{70 kg}{1} \times \frac{1 mg}{1 kg} = 70 \text{ mg}$$

3. How many tablets is that?

$$\frac{70 mg}{1} \times \frac{1 tablet}{20 mg} = 3.5 \text{ tablets}$$

The Parkland Formula

Lastly let's have a look at the Parkland formula. The Parkland formula is commonly used to calculate the amount of fluids to give to burn patients. It is calculated by multiplying the patient's weight in kilograms by the integer of the percent of the body surface area burned (BSAB) by 4mL/kg:

$$\text{PF} = \frac{4 mL}{kg} \times \frac{kg}{1} \times \text{\% BSAB integer}$$

As you can see, the kilos cancel, the integer from the % BSAB is unit less, and we are left with mL. This is the total volume of fluid to administer in the first 24 hours following the burn; the first half of the fluid is administered in the first 8 hours, and the remaining half of the fluid is given over the next 16 hours. Let's assume a 176-lb patient, with 40% BSAB, and calculate drip rates for the 1^{st} and 23^{rd} hours using a 10-gtt set.

1. Figure out the total volume of fluids to give. All we do is plug the numbers - the Parkland formula is, after all, a formula:

$$\frac{4mL}{kg} \times \frac{80kg}{1} \times 40 = 12{,}800 \text{ mL}$$

2. Figure out the drip rate for the first 8 hours. To start, remember that half goes in during the first 8 hours, and 12,800 mL/2 = 6,400 mL. To get to 1 hour we simply divide that by 8, giving us 800 mL/hr.

$$\frac{800mL}{1hr} \times \frac{1hr}{60\min} = 13.3 \text{ mL/min}$$

   And converting that to drops per minute:

$$\frac{13.3mL}{\min} \times \frac{10gtts}{mL} = 133 \text{ gtts/min.}$$

3. Figure out the drip rate for the last 16 hours. For the duration of this tutorial I've been making us do the math the long way; we've earned a break. We can see intuitively that it should be half the rate we used during the first 8 hours, since we are infusing the same volume in twice the time. So what's half of 133? 66.5 gtts/min.

